Operations Research Qualifying Examination
Department of Statistics and Operations Research
——————— August, 2017 ———————

OR Deterministic Qualifying Examination

Information:

- **Student's full name:** ____.
- **Student's signature:** __________________________.
- **Date:**......../...../2017. **Time:** ....
- **Honor pledge:** I have neither given nor received unauthorized assistance on this exam.

General instructions:
Please carefully read the following instructions before writing your answers.

a. Write your name on this sheet. Date and sign on the first page.

b. This examination is closed-book, and consists of two questions.

c. Answer all these questions as clearly and concisely as you are able.

d. Use of the internet and/or mobile devices is not permitted.

The exam questions

Question 1: (60 points) We are given two undirected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$.

a. Assume $|V_1| = |V_2| = n$ and $|E_1| = |E_2| = m$. Describe an integer programming (IP) problem, which is feasible *if and only if* G_1 and G_2 are isomorphic, i.e. there exists

$$f : V_1 \to V_2 \text{ one to one mapping}$$

such that $(i, j) \in E_1 \Leftrightarrow (f(i), f(j)) \in E_2$.

The IP should have 0–1 variables, and a polynomial number of variables and constraints in n and m.

Carefully explain in words the meaning of the variables and constraints. Especially carefully explain the meaning of the logic of the constraints.

b. For this part, we need the concept of an *induced subgraph.* If $G = (V, E)$ is an undirected graph, then the graph (W, F) is an induced subgraph if $W \subseteq V$, $F \subseteq E$ and $i, j \in E$, $i, j \in W$ implies $(i, j) \in F$. That is, if we choose two endpoints of an edge, then we have to choose the edge as well.

Now do not assume that G_1 and G_2 have the same number of nodes, or edges. Describe an IP with 0–1 variables to choose a maximum cardinality induced subgraph of G_1 and of G_2 which are isomorphic.

The IP again should have a polynomial number of variables and constraints.

Again, carefully explain in words the meaning of the variables and constraints. Especially carefully explain the meaning of the logic of the constraints.

Question 2: (40 points) Consider the following linear programming problem:

$$\begin{array}{lllllll} \text{minimize} & 16x_1 + 12x_2 + 10x_3 + 11x_4 & & & \\ \text{subject to} & 180x_1 + 120x_2 + 90x_3 + 60x_4 - x_5 & = 90 \\ & 3x_1 + 2x_2 + 6x_3 + 5x_4 + x_6 & = 4 \\ & x \geq 0 \end{array}$$

a. Verify that the basis $\{x_1, x_6\}$ is an optimal basis, and write down the set of all optimal solutions.

b. Let c_1 be the coefficient of x_1 in the objective function. Its present value is 16. For what range of values of c_1 does the basis $\{x_1, x_6\}$ remain optimal, assuming that all other data entries are fixed at their present values? Moreover, for what values of c_1 within this range does the problem have multiple primal optimal solutions? Write down the set of primal optimal solutions when c_1 takes those values.

c. Let b_1 be the right hand side of the first constraint. Its present value is 90. For what range of values of b_1 does the basis $\{x_1, x_6\}$ remain optimal, assuming that all other data entries are fixed at their present values? Moreover, for what values of b_1 within this range does the dual problem have multiple optimal solutions? Write down the set of dual optimal solutions when b_1 takes those values.

——— The end ———