STOR 642
Comprehensive Written Examination
9:00am-1:00pm, August 13, 2015

This test consists of four questions.
This is a closed book exam. No communication devices of any kind are allowed.
Explain your answers in detail.
The problem and subproblem weights are given in parentheses.

Problem 1. (25 points)

1. (7) A queueing system consists of N service stations, each with a single server. Customers arrive from outside, visit various stations in the system and then leave. What assumptions are needed to model this as an open Jackson network? Introduce the needed notation. Assume these assumptions hold in answering the next three sub-questions.
2. (6) State the traffic equations for computing the total arrival rate to each node.
3. (6) State the stability condition.
4. (6) State the limiting distribution of the state of the system assuming stability.

Problem 2. (25 points)

1. (2) Define a renewal process $\{N(t), t \geq 0\}$.
2. (8) State and prove the almost sure version of the elementary renewal theorem. (Assume strictly positive and finite expected inter-renewal times.)
3. (5) Define $B(t)$, the remaining lifetime at time t in a renewal process.
4. (5) Derive a renewal type equation for $E(B(t))$.
5. (5) Compute $\lim_{t\to\infty} E(B(t))$.

Problem 3. (25 points)

1. (5) Let $X(t)$ be the number of customers at time t in a $G/M/2/2$ system, with common continuous inter-arrival time cdf $A(\cdot)$, and iid $\exp(\mu)$ service times. Let S_n be the time of the n-th arrival (not necessarily entry). Assume $S_0 = 0$. Let $X_n = X(S_n-)$. Using this, show that $\{X(t), t \geq 0\}$ is an Markov Regenerative process.
2. (7) Compute the kernel $G(\cdot)$ of $\{X(t), t \geq 0\}$.
3. (4) Compute the transition probability matrix of $\{X_n, n \geq 0\}$.

4. (6) Compute $\tau_{i,j}$, the expected time spent by the $\{X(t), t \geq 0\}$ process in state j over $(S_n, S_{n+1}]$ given that $X_n = i$.

5. (3) Show how to compute the limiting distribution of $X(t)$ as $t \to \infty$ using the above quantities.

Problem 4. (25 points)

1. (4) Define a Brownian motion with drift parameter μ and variance parameter σ. Define a standard Brownian motion.

2. (7) Let $\{B(t), t \geq 0\}$ be a standard Brownian motion. Show that $B^2(t) - t$ is a Martingale.

3. (7) Derive the distribution of $\int_0^t B(u)du$.

4. (7) Compute the mean and variance of $\int_0^t B(u)dB(u)$.