STOR 641
Comprehensive Written Exam
August 16, 2012

This test consists of two questions.
This is a closed book exam.
Explain your answers in detail.
The duration of the exam is two hours.
The relative weights are given in the parentheses.

Problem 1. A machine produces items at a rate of one per day and are stored in a warehouse. The items are perishable with a fixed lifetime of d days, that is, if an item does not get used within d days after production, it is discarded. Let D_n be the demand on day n, and assume that $\{D_n, n \geq 0\}$ is a sequence of iid random variables with

$$a_i = P(D_n = i), \quad i = 0, 1, 2, \cdots.$$

Assume that the production occurs at the beginning of a day, and the demand occurs at the end of a day, and the manager uses the oldest items first to satisfy the demand. Any unsatisfied demand is lost.

1. (2) Give the definition of a general time homogeneous DTMC on state space $\{0, 1, 2, \cdots\}$.
2. (4) Model the system described above as a DTMC. What is its state space? What is the transition probability matrix?
3. (4) Does the limiting distribution exist? Why or why not? Write the balance equations and the normalizing equation. Derive a recursive method of computing the limiting distribution that avoids solving simultaneous equations.
4. (3) Now consider a special case where $a_i = 0$ for $i > 2$. Write an explicit expression for the limiting distribution.
5. (3) Compute the average age of the items in the warehouse in steady state.
6. (3) Compute the average age of the items that are issued to satisfy demand on a day on which there is a positive demand (in steady state).
7. (2) Compute the fraction of items discarded due to old age.
8. (4) Formulate a DTMC if the manager uses the newest items first to satisfy the demand.

Problem 2.

1. (1) State the memoryless property of a continuous non-negative random variable.
2. (3) Show that a continuous random variable has memoryless property if and only if it is an exponential random variable.
3. (2) Give one definition of a Poisson Process.
4. (5) Suppose customers arrive at a single server service station according to a PP(λ). The service times are iid Exp(μ). When the number in the system reaches a fixed number $K > 0$, all the customers are shipped off instantly to another location and the system becomes empty, and the cycle repeats. Model this as a CTMC by giving its state-space and generator matrix.

   Answer the following questions for the system in part 4.

5. (5) Compute the limiting distribution of the state of the system.
6. (5) Suppose the cost of keeping a customer in the system is h per person per unit time, while the cost of shipping off the customers to another location is s per customer. Compute the long run cost per unit time as a function of K.
7. (4) Compute the expected time between two consecutive epochs when the customers are shipped off to the other location.