August, 2012 Name:____________________

COMPREHENSIVE WRITTEN EXAM – STOR655 MATHEMATICAL STATISTICS

All problem parts have equal weight. In budgeting your time expect that some part will take longer than others. When solving multi-part problems feel free to use results of earlier parts even if you cannot solve them in proving later parts. Do not forget to split the time between both STOR654 and STOR655 papers.

1. Let $X_1, \dots, X_n$ be i.i.d. $U(\theta, 2\theta)$, $\theta > 0$.

   (a) Find $\hat{\theta}_{\text{MLE}}$ the MLE of θ.

   (b) Find $\hat{\theta}_{\text{MM}}$ the MM of θ.

   (c) Assuming an improper prior $\theta \sim \theta^{-1} I_{(0,\infty)}(\theta)$ find $\hat{\theta}_{\text{B}}$ the posterior Bayes estimator of θ (use square loss).

   (d) Find the asymptotic distribution of θ_{MLE}. (Hint: Start by finding a_n so that $a_n(\theta_{\text{MLE}} - \theta) \xrightarrow{\mathcal{D}} Y$, where Y is non-degenerate.)

   (e) Find the asymptotic distribution of θ_{MM}.

   (f) Find the asymptotic distribution of θ_{B}.

   (g) Based on your asymptotic calculations, which of the estimators would you prefer? Comment on asymptotic efficiency.

2. Let $X_1, X_2, \dots, X_n$ be i.i.d. $N(\mu, \sigma^2)$. Assuming the improper prior $(\mu, \sigma^2) \sim \sigma^{-2}$ find the 95% credible interval for μ. Is the interval you found a 95% confidence interval?

3. Let $X_1, \dots, X_n$ be i.i.d. Poisson(λ_x) and $Y_1, \dots, Y_n$ be i.i.d. Poisson(λ_y); X's are independent of Y's.

   (a) Find the GLR, Λ_n for testing $\mathcal{H}_0 : \lambda_x = \lambda_y$ versus $\mathcal{H}_1 : \lambda_x \neq \lambda_y$. What is the asymptotic distribution of $-2 \log \Lambda_n$ under $\mathcal{H}_0$?

   (b) Assume that $\lambda = \lambda_x = \lambda_y$. Find the asymptotic variance of the MLE of λ,