STOR 762
Comprehensive Written Exam
August 14, 2013

- This exam consists of 3 questions on 2 pages including this cover page.

- This exam is closed book and closed notes.

- You are not allowed to use a calculator or a cell phone throughout the exam.

- Explain your answers in detail to receive partial credit.

Problem 1. (35 points) Suppose that you will estimate the integral $I = \int_0^1 g(x)dx$ by means of Monte Carlo simulation.

(a) Explain step by step how you can apply the antithetic variates (AV) approach to obtain $\hat{I}$, which is the Monte Carlo point estimator for I. Suppose that you use $\{U_i\}_{i=1}^n$, which is a stream of n random numbers.

(b) If $g(x) = x^2$, then what is the variance of the estimator $\hat{I}$ under AV?

(c) If $g(x) = x^2$, then what is the variance of the estimator $\hat{I}$ under independent sampling, i.e., without using the AV method? Obtain the percentage reduction in variance achieved by the AV method over independent sampling.

Problem 2. (30 points) Suppose that arrivals to a service center follows a non-stationary Poisson process with rate function $\lambda(t) = 1/(t+a)$, for time $t \geq 0$ and some positive constant a. Your goal is to use a sequence of random numbers $\{U_j\}_{j \geq 1}$ to construct a sample-path for this non-stationary Poisson process. In particular, provide an expression for S_i, which denotes the arrival time of the ith customer for $i = 1, 2, \ldots$, as a function of the given sequence of random numbers.

Problem 3. (35 points) To compare the performance of two system configurations, each system was simulated using the multiple replications method with $n \geq 2$ replications. For the simulation of the two systems, common random numbers were used. Using the replication means for system 1, a 95% confidence interval on the mean performance was obtained as $\bar{X} \pm t_{n-1,0.025}\sqrt{\hat{V}_X/n}$, where $t_{n-1,0.025}$ is the 97.5% quantile for the t-distribution with $n-1$ degrees of freedom. Similarly, a 95% confidence interval on the mean performance of system 2 was obtained as $\bar{Y} \pm t_{n-1,0.025}\sqrt{\hat{V}_Y/n}$. It turned out that $\bar{X} < \bar{Y}$ but we want to know whether the difference between the two means is statistically significant or not.

(a) One way to determine whether these two systems are statistically different or not is to check whether the two confidence intervals given above overlap or not. If you apply this approach to make a conclusion about the experiment described in the problem statement, then what is the condition under which the two systems can be declared statistically different?

(b) Another way to compare the two systems is to apply the paired t-test to the data collected. If you apply this approach to the simulation experiment described in the problem statement, what would be the condition under which your conclusion is that the two systems are statistically different at $\alpha = 0.05$ level of significance?

(c) Which one of the above approaches is more powerful in detecting statistical significance at the same level of significance with the same data? Justify your answer based on your answer to parts **(a)** and **(b)**.