STOR 642
Comprehensive Written Examination
Thursday, August 18, 2011

Write your assigned letter on each sheet of your answer book. Do not write your name anywhere.
This test consists of three problems.
All problems carry equal weight.
This is a closed book exam.
Explain your answers in detail.
The duration of the exam is two hours.

Problem 1. Jobs arrive at a service facility according to a Poisson process with rate λ. There is a single server. The probability that the service of a job is carried out successfully is p independently of the other jobs, the arrival and service processes. If the service is not successful, the job needs to go through service again. Service of each job (a repeat or not) takes an exponential amount of time with mean $1/\mu$ independently of everything else. The manager is considering two different system designs: In System 1, once the server picks up a job, it will stick with that job until it is served successfully. In System 2, if service is not successful, the job in service joins the queue again from the back.

(a) Suppose that the manager wants to adopt the design under which the expected number of jobs in the system in the steady state is smaller. State the stability conditions, compute the expected number for both systems and make a recommendation.
Hint: What is the distribution of the total service time of a single job?

(b) Suppose that the manager decides to go with System 1. Now, the manager is considering replacing the current server by a new one. Service times under the new server are again exponentially distributed with mean $1/\mu$. The expected value of the number of times each job needs to go through service is $1/p$ while its variance is σ^2. Under what condition would the manager like to go with the new server given the objective of minimizing the expected number of jobs in the system?

Problem 2. A single server works on an infinite supply of jobs. The amount of time it takes the server to work on a single job is exponential with rate μ, successive processing times being iid. If the service of a job takes less than b units of time, the server immediately starts on a new job. Otherwise, it takes a break for $a > 0$ units of time and then starts a new job immediately afterwards. Suppose that at time zero, the server starts a new job. Let $A(t)$ denote the state of the server at time t. If the server is working at time t, we let $A(t) = 1$, otherwise we let $A(t) = 0$.

(a) Let $t > b$ and derive a renewal-type equation for $P\{A(t) = 1\}$.

(b) Use Key Renewal Theorem to compute $\lim_{t\to\infty} P\{A(t) = 1\}$.

Problem 3. Consider a stable $G/G/1$ queue with i.i.d. service times and i.i.d. interarrival times. Let $X(t)$ denote the number of customers in the system at time t and let $Z(t)$ denote the number of customers immediately after the last departure before time t.

(a) Is $\{X(t), t \geq 0\}$ (i) a regenerative process, (ii) a semi-Markov process, (iii) a Markov regenerative process? Is $\{Z(t), t \geq 0\}$ (i) a regenerative process, (ii) a semi-Markov process, (iii) a Markov regenerative process? Clearly explain why or why not. State any initial conditions you need if necessary.

(b) Answer part (a) assuming that it is an $M/G/1$ queue.

(c) Answer part (a) assuming that it is an $M/M/1$ queue.