STOR 665 CWE 2023

Student Name:

Theory Part

1. Consider the distribution with density $f_\lambda(x) = x\lambda^2 \exp(-\lambda x), \lambda > 0$

(a) Write the distribution in the canonical form of the exponential family and obtain the canonical parameter, the function $b(\cdot)$, the function $c(\cdot, \cdot)$ and the dispersion parameter ϕ.

(b) Obtain the variance function, the deviance residuals and the Pearson residuals for a GLM where the responses y_i have distributions with densities f_{λ_i} .

2. Assume we wish to compare three anti-epileptic drugs A, B and C. The outcome is the number of seizures observed for a patient diagnosed with epilepsy in a given time period; this number can be 0 or a large integer. It is known that age of the patient and sex of the patient are important predictors, and it is suggested that the effect of age may differ by sex of the patient. Assume patients are randomly assigned to a treatment where they receive one of the drugs. The covariates available for each patient are age (continuously recorded), sex of the patient (f or m) and type of drug assigned to the patient (A, B or C). We wish to model the effect of the covariates on the number of seizures recorded and it is of specific interest to determine whether any of the drugs have beneficial effects in reducing the number of seizures.

(a) Define the predictor variables and specify all components of a GLM. (Please use drug C as the baseline category.)

(b) Assume one wants to find out whether indeed the effect of age differs by sex. How do you proceed? Please write the null hypothesis and alternatives of the test, provide the form of the test statistic, and how to determine whether the null should be rejected or not.

(c) The investigator wants to establish that the three drugs do not have the same effect. How do you proceed? Please write the null hypothesis and alternatives of the test, provide the form of the test statistic, and how to determine whether the null should be rejected or not.

(d) The investigator informs you that there may be a subgroup of patients, who were included because they had a seizure a long time ago, but are actually not epileptic anymore. For these patients, there is no potential to observe any seizure, however we don't know their identity. Extend your model accordingly and provide details of the extended model.